

Hint: A MATLAB/Octave function can take a handle to another function as a parameter, e.g.

```
function x=foo(f,x0)
```

where  $f$  is a function handle. The function handle can then be called the same way as a function, e.g.  $y=f(x_0)$ .

You can pass a function directly as an argument by using a lambda expression:

```
foo(@(x) x+1, 0)
```

or you can write the function in an m-file, e.g. `myfunc.m`, and refer to it as follows:

```
foo(@myfunc, 0)
```

45. Write a program in MATLAB or Octave which takes as input parameters:

- a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ ,
- its Jacobian  $Df : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ ,
- a starting point  $x_0$  (an  $n \times 1$  vector) and
- a tolerance `tol`

and performs **Newton's method** (without line search) for the equation  $f(x) = 0$  (up to the tolerance `tol` and starting at  $x_0$ ).

46. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = \arctan(x)$  (in radians).

- (a) What is the solution  $\hat{x}$  of  $f(x) = 0$ ?
- (b) What is the derivative  $f'(x)$ ?
- (c) Apply your program from example 45 to this example with  $x_0 = 2$  and the tolerance  $\sqrt{\text{eps}}$ . Does it converge? Why?

47. Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ ,  $f(x_1, x_2) = \begin{pmatrix} x_1 - x_2 \\ x_1 x_2 \end{pmatrix}$ .

- (a) What is the solution  $\hat{x}$  of  $f(x) = 0$ ?
- (b) What is the Jacobian  $Df(x) = f'(x)$ ?
- (c) Apply your program from example 45 to this example with  $x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$  and the tolerance  $\sqrt{\text{eps}}$ . Does it converge? How many iteration steps does it take?

48. Write a program in MATLAB or Octave which takes as input parameters:

- a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ ,
- its Jacobian  $Df : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ ,
- a starting point  $x_0$  (an  $n \times 1$  vector) and
- a tolerance `tol`

and performs **Newton's method with Armijo line search** (i.e. the "modified Newton method" from the lecture transcript) for the equation  $f(x) = 0$  (up to the tolerance `tol` and starting at  $x_0$ ).

49. Apply your program from example 48 to example 46. Does it converge? How many iteration steps does it take? What can you conclude?

50. Apply your program from example 48 to example 47. Does the line search make any difference in this example? Why?
51. Write a program in MATLAB or Octave which takes as input parameters:
- a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ ,
  - a starting point  $x_0$  (an  $n \times 1$  vector) and
  - a tolerance `tol`

and performs **Broyden's rank 1 method** (with Armijo line search, as in the lecture transcript) for the equation  $f(x) = 0$  (up to the tolerance `tol` and starting at  $x_0$ ).  
Note: Broyden's method does not need the Jacobian.

52. Apply your program from example 51 to example 46. Does it converge? How many iteration steps does it take? Compare the efficiency of the algorithm with the Newton algorithm with the same Armijo line search (example 48, example 49).
53. Apply your program from example 51 to example 47. Does it converge? How many iteration steps does it take? Compare the efficiency of the algorithm with the Newton algorithm with the same Armijo line search (example 48, example 50).